Single spheroid settling: a benchmarking data-set generated by spectral/spectral-element DNS

A full description of the simulations is available in reference [6].

Description of the flow

A single oblate spheroid (rigid, with homogeneous mass density) is freely settling under the influence of gravity in an ambient, viscous, incompressible fluid.

Geometry

The computational domain is cylindrical, with the cylinder axis aligned with the direction of gravity. The particle is located on the axis of the cylinder, at a distance L_u from the inflow and at L_d from the outflow plane (cf. sketch in figure 1 and table 1). The particle has an equatorial diameter d, and an aspect ratio $\chi = d/a$, where a is the length of the symmetry axis.

Flow parameters

The problem is governed by three parameters. The first is the particle's aspect ratio χ; the second is the solid-to-fluid density ratio ρ_p/ρ_f; the third is the Galileo number $G = u_g\, d/\nu$, where $u_g = (|\rho_p/\rho_f - 1|\, |\mathbf{g}|\, V_p/d^2)^{1/2}$ is the gravitational velocity scale, $V_p = \pi d^3/(6\chi)$ the particle's volume, $\mathbf{g}$ the vector of gravitational acceleration and ν the kinematic viscosity of the fluid. Two aspect ratios were considered, the density ratio was varied between 2.1 and 14.3, the Galileo number was varied in the range from 100 to 220 (cf. table 2), leading to four different flow regimes and patterns of particle motion.

- Case "A": steady vertical;

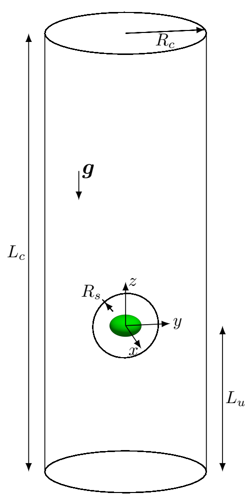

Figure 1: The geometry of the problem and the computational domain.

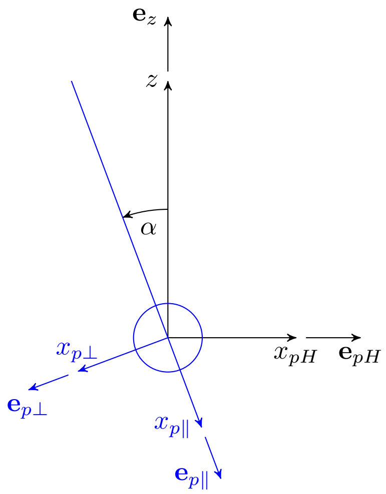

Figure 2: Sketch of the notation concerning directional unit vectors and coordinates in the plane spanned by the vertical axis $\mathbf{e}_z$ and the direction of the particle motion $\mathbf{e}_{p\|}$, as defined in [6; 7]. Note that the direction $\mathbf{e}_{pHz\perp}$ is perpendicular to the plane of the sketch which corresponds to the plane defined by the trajectory and the vertical.

R_c	L_c	L_u
2.67	15	5

Table 1: Dimensions of the cylindrical domain: radius R_c, length of the domain L_c, and distance from the sphere's center to the upstream boundary L_u.

case	χ	Ga	ρ_p/ρ_f
A11	1.1	100	2.1
B11	1.1	115	2.1
B15	1.5	110	2.14
C15	1.5	150	2.14
D11	1.1	200	2.1
D15	1.5	220	14.32

Table 2: Chosen parameter points.

- case "B": steady oblique;
- case "C": vertical periodic;
- case "D": chaotic.

Numerical method and resolution

The method solves the incompressible Navier-Stokes equations on a mesh which is translating with the spheroid, thereby avoiding remeshing. The spatial discretization uses a truncated Fourier series in the azimuthal direction and a spectral-element approach in the radial/vertical plane. The spheroidal particle is embedded in a mesh with spherical outer boundary, which in turn rotates with respect to the cylindrical background mesh; matching at the interface is performed with the aid of spherical harmonics. The temporal discretization is semi-implicit using a third order Adams-Bashforth method for the non-linear terms. Momentum and continuity equations are uncoupled through a projection method. Details can be found in [1–4].

- Grid: $\chi = 1.1$ features 194 elements; $\chi = 1.5$ has 190 elements.
- 6 collocation points in each of the two spatial directions internal to each element.
- Truncation of azimuthal Fourier series at wave-number 7.
- Time step: such that $CFL \approx 0.25$.

Boundary conditions

- inflow: zero (ambient) velocity.
- side-walls and outflow: no stress Neumann condition
- pressure: homogeneous Dirichlet condition on all Neumann boundaries of velocity.

Available data

The fluid velocity data is presented relative to the particle velocity. A Cartesian coordinate system attached to the particle center is used which is constructed as follows. The "local trajectory plane" (shown in figure 2) is defined by $\mathbf{g}$ and by the particle velocity vector; the latter one defines the unit vector $\mathbf{e}_{p\|}$. The vector oriented normal to that plane is denoted by $\mathbf{e}_{pHz\perp}$. The two unit vectors $\mathbf{e}_{p\|}$ and $\mathbf{e}_{pHz\perp}$ are completed by $\mathbf{e}_{p\perp}$ which lies in the trajectory plane and which is perpendicular to the particle trajectory.

The data-set contains the following data items:

- relative velocity deficit on the axis through the particle;
- cross-profiles of the three (relative) velocity components at 4 distances downstream of the particle, taken along the directions $\mathbf{e}_{pHz\perp}$ and $\mathbf{e}_{p\perp}$;
- pressure coefficient on great circle;

- temporal evolution of the different particle degrees of freedom (time-periodic case “C” only);
- probability density function and temporal auto-correlation of the particle degrees of freedom (chaotic case “D” only).

Please note that all lengths given herein and in the data-base are **normalized with the particle diameter** d; all velocity data is **normalized with the gravitational velocity** u_g.

Data format and location

Data is presented in the form of “mat-files”* (readable e.g. with Matlab and GNU/octave). A script for reading the data (and plotting it) is provided. The data is located below the following URLs:

www.ifh.kit.edu/dns_data/particles/single_spheroid_sedimentation
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*A detailed description of the file format can be found here: http://www.mathworks.com/help/pdf_doc/matlab/matfile_format.pdf